

Project Success: Matters Arising

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(Submitted: June 30, 2006; Accepted: December 27, 2006)

Abstract

A project is assumed to be successful if it adheres strictly to the set criteria of quality, cost and schedule. While difficulties in measuring and assessing quality is minimal, there are a lot of problems associated with that of time and cost as different models are utilized in estimating these parameters sometimes with little effort at integration. This makes it rather difficult to put in place a meaningful control procedure to curtail the incidence of time and cost overruns. Since time estimate is generally stochastic, it is difficult to state precisely the completion time and its accompanying costs which do not also take into consideration the idle cost which is inevitable in any system due to resource allocation problems. Considering the above fact, project success can only be measured based on an acceptable range. This study employs five stochastic models using a hypothetical case study as all projects surveyed lack the necessary data for proper analysis and this is then used as a platform for analyzing some selected projects as to the extent of their time and cost overruns. This study therefore could help the parties to a project know in advance the likely time and cost the project may be subjected to in order to plan accordingly. The approach has helped the contractors and clients enormously in reducing conflict arising from time and cost overruns in this part of the world and the same could be applicable in other economies.

Keywords: Stochastic models, project success, resource smoothing, time estimate, quality.

1.0 Introduction

Due to the complexity normally associated with time and cost estimation, project success analysis is normally approached in two different directions – the qualitative approach dealing mainly on project critical success factors with notable contributors such as Pinto *et al* (1989) and Baker *et al* (1983). It is however difficult to state precisely the contributions of these critical factors to the aspect of time and cost (the critical factors to project success) using numerical figures from any project under review. To overcome this shortcoming, quantitative approach has dominated the literature with emphasis on network scheduling models and Bar/Gantt chart. This approach is adopted for the present study. Though these techniques are all embracing, it is observed that time, cost and resources have over the years been treated in some fashion as different entities. These approaches have taken four different forms:

- (a) Time estimate in determining project duration and other indices such as earliest starting time, latest starting time, total float, etc.
- (b) Time and resources dealing on such topics as resource loading, resource smoothing/levelling and constrained resource models using different mathematical models and priority rules.
- (c) Time and cost in the form of expenditure control loop.
- (d) Cost estimate in the form of bills of quantities (BOQ).

1.1 Time Dimension

A lot of interest have been shown in the area of time estimation since the emergence of PERT (Program Evaluation & Review Technique) in 1958, a work pioneered by a team of Booz, Allen & Hamilton Consultants and United States Navy in the design, planning and development of Polaris missile project. PERT team utilized mainly beta distribution function in time estimation. Ever since, barrage of criticisms have trailed PERT and its assumptions. There is no evidence, according to Elmagraby (1977) as to why

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beta distribution was chosen in the first instance although he later supported the idea because of its stipulation of three time estimates which is bound to give a better estimate. However, he cast doubts as to the validity of original PERT claim and catalogued a series of improvements which ought to be made in reshaping PERT in order to place it on a more solid foundation. The assumption that all activities are independent which necessitated the use of normal distribution in trying to determine the probability that a certain due date in any project could be met is not true and he went further to prove this using Tippett (1925) model. Looking at the various criticisms of PERT with respect to beta distribution, it may not be entirely out of place if other distribution models are used which could even give a better representation. It is without doubt that attempts in this direction (Nkasu (1983)) have been undertaken with their own limitations too. It is on the basis of this fact that five distribution models would be used in the analysis of the hypothetical case study used for this work at a later stage.

1.2 Time and resources

Time has no intrinsic value if it is not related to the number of resources (labour and other transforming inputs) and/or efficiency in which it is based on. Having determined the duration of the project, it is always normal to find out in total for each time period (e.g. hours, day, weeks, etc.), the number of resources needed to support the implementation programme. In the case of network scheduling technique with no resource constraint (unconstrained resource model), resource loading is carried out using either the earliest starting time or latest starting time whereas in the case of the Bar/Gantt chart, the exercise is mainly based on the earliest starting time. It must be realized that the Bar/Gantt chart is incapable of producing the latest time schedule in its original form. There would therefore be peaks and valleys associated with these schedules and if the loading is based on the earliest starting time, attempts at having one schedule which is optimal may be contemplated. This is achieved by what is normally termed resource smoothing/levelling by moving activity (or activities) with floats from the high peak regions to be started at a later date and it is assumed that as this is done, the valleys will be filled to smooth the resource profile subject of course to project duration as determined by the critical path. If on the other hand, the resources are indeed limited, the objective would shift to finding the minimum project duration which may likely be beyond that of the critical path, that means finding a minimum path of a maximal problem. This is usually achieved by the use of mathematical models such as linear programming (Pritsker (1969)), branch and bound algorithms (Hastings (1972)) and priority rules (Davis (1974) and Russel (1986)). Over the years, several solutions employing mathematical techniques have appeared in the open literature to deal with this resource- duration minimization problem but a lot of attention have now shifted to the use of priority rules because of their simplicity and the ability to handle a more practical problem devoid of a lot of assumptions. However, there have been conflicting claims as to the superiority of the different priority rules. Having compared different priority rules, Akpan (2000) has found the use of random activity selection (RAS) as having an edge over all others and this priority rule would be used later in the course of this work. It would be appreciated that it is not possible to hire and fire resources (labour (fixed and variable), equipments, etc) at will, a certain minimum level has to be maintained throughout a reasonable period of time or the entire project duration. With reality comes idle capacity with its attendant cost which is inevitable in any system.

1.3 Time and Cost

The optimization problems as enumerated above have everything in common with cost. While the constrained and unconstrained resource optimization models concentrate on the resources as prime causes of cost which have to be minimized by opting for the schedule which makes this possible, this section of consideration treat cost as a convenient homogeneous measure of resources which act or interact to cause the requisite activities to be completed. Even though time/cost trade off analysis comes under a different category, it would however not merit any attention here since it is not of general application. Generally, cost is assigned to individual activities and thereafter loaded using either the earliest starting time or latest starting time. If the exercise is carried out in these two extremes then separately drawn on a cum

basis, one would end up with expenditure control loop which would indicate possible expenditure pattern likely to be experienced during project execution. This technique could also serve as a control tool as expenditure outside the loop signals possible dangers of either cost or time overruns or both.

1.4 Cost Dimension

Here cost is the dominant factor normally presented in the form of bills of quantities (BOQ). Costs are assigned to individual work packages without any attempt of stating when each work package would be carried out during project execution. According to Akpan *et al* (2001), in the event of delay the BOQ cannot provide the basis for the estimation of that delay and the cost associated with it. Analysis of cost deviation is always difficult to undertake since the different costs are usually lumped together.

2.0 Observation

From the foregoing, it is observed that the term "project success" is loaded with a lot of uncertainties as to time and cost. While it is simple to estimate to some extent the material and overhead costs, it is difficult indeed to state precisely the project completion time and cost of the transforming inputs. It is on the basis of this that one should be talking of a range of possible completion times and costs since the elements making up these two factors are probabilistic in nature.

3.0 Resource and Time Estimation Using Stochastic Models

As earlier stated, there are a lot of problems associated in estimating time and cost as different models are utilized in estimating these parameters. With respect to time, five probability distribution models – Beta, Exponential, Normal, Uniform and Weibull (see Appendix A) are used for this purpose which indirectly has an effect on cost. Since it is difficult to get a field data (i.e. those from the construction industry) encompassing all the materials as outlined in section 1, a hypothetical project consisting of 37 activities (including dummies since the project adopts Activity-on-Arc approach) utilizing four resources designated as A, B, C and D is used for the exercise. Three parameters are specified as these are required by some of the stochastic models and the information are shown in Table 1. The costs of using each resource for a week are ₦120, ₦150, ₦100 and ₦80. The analysis is based on both deterministic model of CPM and probabilistic time using the five stochastic models.

The minimum time would be the basis used to determine optimality in line with the general project management objective and each model is subjected to 30 iterations and the schedule with the minimum time chosen as the optimum in line with the general constrained-resource problems. In the course of the research, it was found that two or more schedules may have the same time but different cost thus making it difficult to choose the schedule with the absolute minimum. To discriminate among such schedules, the cost is discounted at 11% on a 52-working period in a year taking into consideration the beginning and ending of each activity and using a uniform continuous discounting over the domain. This is expressed as :-

$$Q = \int_a^b C e^{-jt} dt$$

where j is constant over time and Q is the discounted amount.

Looking first at the unconstrained resource model and taking the most likely time estimates (i.e. Mt) as the activity duration, we have a project duration of 104 weeks. The cost and level of resources at these two extremes (earliest starting time and latest starting time) are ₦813100 (₦905840 when not discounted) and ₦660018 (₦735280 when not discounted) with resources levels of 35, 15, 13, 15 and 26, 13, 12, 10 (all in units) for resources A, B, C, and D respectively. To take care of the constrained resource

situation and for the fact that organizations may not be in a position to secure all the resources as dictated by critical path loading, maximum levels of resources A, B, C, and D are kept at 8, 7, 7, 9 respectively. Based on deterministic activity duration model, a project completion time and cost of 168 weeks and ₦484971 respectively are realized with a weighted resource utilization of 55%, 45% representing the idle cost amounting to ₦217506. Table 3 shows the summary of project completion times and cost together with their weighted resource utilization associated with each distribution model.

In some cases, however, minimum cost may come from those schedules with longer time duration. Total cost is based on the maximum resource levels actually in use throughout the project duration.

The project duration lies between 166 weeks coming from Beta distribution and also with the lowest cost of ₦457755.00 and 185 weeks of Weibull distribution and also experiencing the highest cost of ₦500619.00. There is therefore a time span of 19 weeks and a cost difference of ₦42864.00 representing 11.45% and 9.36% of time and cost respectively with respect to the lower bound.

It must be noted that the completion time is a function of individual activity times, maximum level and intensity of usage and above all, the activity selection at each node when more activities are due for consideration. From the analysis it is observed that Weibull distribution seems to give the pessimistic results in parameter estimation while beta and exponential seem to give the most likely estimate. The reason for this lies mainly in the nature of each distribution model and the underlying parameters used to describe them. The beta distribution requires three parameters, the optimistic, the most likely and the pessimistic to estimate each variate. Since more weight is given to the mode (that is, the most likely with 2/3 of the value), the total activity times are very close to those derived from the deterministic model which is based on the most likely time estimate of the activity.

The exponential distribution requires only a single parameter and variates simulated from it bears only one relationship in the sense that they have to stay within the specified range.

Variates simulated from Normal distribution require the mean, μ and standard deviation, σ of these variates to be specified. The degree of variability which is represented by the standard deviation dictates the extent to which the variate will spread. The larger the standard deviation, the wider the spread and vice versa.

Uniform distribution being rectangular in shape requires two parameters to simulate the variate and these correspond to the optimistic and pessimistic estimates as used in the model.

The Weibull distribution is a special distribution of the gamma and exponential type. It requires two parameters, the scale and the shape parameters although sometimes the third, the location parameter is needed to simulate its variates. When the shape parameter is equal to one, the formula becomes exponential probability density function.

Table 1 : Data relating to the hypothetical case study

Activity	TIME			RESOURCES					
	Nodes		Optimistic(Ot)	Most likely(Mt)	Pessimistic(Pt)	A	B	C	D
	From	To							
1	1	2	1	2	3	3	0	2	1
2	1	9	19	23	27	2	2	0	1
3	1	3	2	4	6	2	3	6	1
4	2	4	1	3	5	3	0	5	1
5	2	10	9	12	15	2	4	2	0
6	3	4	0	0	0	0	0	0	0
7	3	9	4	6	8	3	4	1	2
8	4	5	2	4	6	2	0	1	3
9	5	9	7	10	13	3	4	2	2
10	5	6	2	3	5	2	1	2	0
11	6	8	20	24	27	3	3	0	1
12	6	7	2	3	4	3	0	5	3
13	7	8	10	12	14	2	0	3	2
14	8	10	1	2	3	2	3	1	0
15	9	10	12	15	18	3	3	2	2
16	9	19	22	25	30	4	0	3	0
17	10	15	14	17	19	3	4	0	2
18	10	11	1	2	3	3	2	0	4
19	10	19	8	10	13	7	1	2	0
20	11	12	1	2	4	2	2	3	4
21	11	19	6	8	10	3	0	2	5
22	12	14	1	1	2	6	0	0	2
23	12	13	1	2	3	3	1	3	2
24	13	14	0	0	0	0	0	0	0
25	13	17	5	7	10	3	0	2	1
26	13	18	8	11	13	2	2	0	3
27	13	20	15	18	20	2	1	2	1
28	14	15	1	1	3	2	3	0	2
29	14	16	1	2	3	7	2	2	1
30	15	16	2	3	5	2	2	1	2
31	16	17	1	2	4	3	2	0	3
32	17	18	25	28	32	3	7	1	0
33	18	22	13	16	18	4	2	1	0
34	19	20	14	16	20	2	3	2	3
35	20	22	15	17	21	3	3	2	0
36	19	21	11	12	14	4	4	2	2
37	21	22	28	31	35	3	2	5	0

Table 2 : Variates generated from the different distribution models

Activity	Beta	Exponential	Normal	Uniform	Weibull
1	2	2	2	3	3
2	20	20	26	21	25
3	3	5	4	6	6
4	4	2	5	2	5
5	13	14	13	12	15
6	0	0	0	0	0
7	7	8	7	7	8
8	5	6	5	5	6
9	9	10	11	13	12
10	3	3	4	2	5
11	23	24	25	22	27
12	3	2	4	2	4
13	13	13	13	12	12
14	2	3	2	3	3
15	16	14	16	13	16
16	25	25	28	25	28
17	18	17	18	17	15
18	2	2	3	3	3
19	11	9	12	13	11
20	3	2	3	2	4
21	8	8	8	8	10
22	1	2	2	2	2
23	2	2	2	3	3
24	0	0	0	0	0
25	8	7	7	7	9
26	10	12	11	11	13
27	18	16	19	19	18
28	2	2	1	1	3
29	2	3	2	2	3
30	3	3	3	4	5
31	2	2	2	3	4
32	28	26	28	26	27
33	15	18	17	18	17
34	16	15	16	16	20
35	15	18	18	20	19
36	12	14	12	11	14
37	31	34	32	33	34

Table 3: Summary of Project Completion Times, Costs and Weighted Resource Utilisation (WRU) associated with each Distribution model

Stochastic Models	Time	Cost (₹)	WRU
Beta	166	457755	59%
Exponential	169	464638	59%
Normal	182	481880	59%
Uniform	172	494547	57%
Weibull	185	500619	61%

5.0 Field Survey

It is an undeniable fact that projects are rarely completed on time and within budget. Though many factors have been attributed as to the causes of these twin problem, what seems to be lacking in the open literature is the acknowledgement of the incidence of unavoidable idle resources which is inevitable in any system as analysed and presented above. It is because of this fact that an enormous amount of work has been carried out to minimize this incidence and one should have expected that this idle cost emanating from them should have been incorporated in the final cost of production. In conventional production systems, it is "naturally" absorbed (e.g. cost of machine waiting to receive an input from preceding machine) but that does not seem to be the case with project management system. Increases in the price of constructional materials are always acclaimed to be the major cause of cost overrun even with no empirical evidence.

To validate some of these claims, a survey presented in Table 4 was carried out for further analysis and the information is extracted from the Visitation Panel Report of Bayero University in Nigeria which exercise took place in 2004. Looking critically at projects 2 and 3, the two having very short period of execution, there could not have been an appreciable increase in the price of constructional material to have warranted 4.70% and 22.71% cost overrun. From the report, there was no increase in the scope of work to have attracted this additional sum. The investigation then shifted to the cost of the transforming inputs. Building on the conjecture that no provision was made for the unavoidable idle cost at the time of tendering, this extra cost is directly a by-product of it. Assuming that the rule of thumb of material-labour ratio of 2:1 as normally applied in the construction industry is followed, the cost overrun should be related only to the transforming inputs following from our argument and for these two projects, the percentage variation would therefore be 13.48% and 55.53% respectively. In a similar vein, the time overrun needs a close scrutiny. The time of carrying out the work as given was rather unrealistic when compared to project 1 in terms of size (using monetary unit as a measure) and the type of work involved where a greater proportion of work would be allowed to properly set before further work can be undertaken. There are however, some projects with no contract variation but an excessive time overrun and even abandonment. Enquiries revealed that these projects were awarded on "fixed price contract" basis (Veld *et al.*, (1989)). Though this contract type is advantageous in reducing conflict between the parties as to price variation, the guarantee that the contractor would not cut corners and deliver substandard work in order to stay within the contract sum is very high indeed. Secondly the possibility of the contractor abandoning the job if the cost goes beyond his expectation is equally high. Following from our discussion, the idle cost associated with labour might have contributed substantially to the variance. To avoid these undesirable consequences, an incentive scheme to take care of the above should be built into this type of contract to help the contractor offset a part of the cost if this situation arises although this should be kept secret from the contractor (so that he may not take advantage of it) rather than allowing the job to be abandoned. However, it is not a norm that projects should normally encounter unfavourable variations as to time and cost but this seems to be the line of thinking on the part of the contractors. The cost variations for all the projects are moderate and may fall within the range of acceptable limit but the time variation in some of the projects are rather on the high side.

6.0 Discussion

Information from the field as to how the completion times of projects are determined was rather shocking. The determination was mainly based on guesstimate and experience, not on the work plan which should have been derived from network scheduling technique. How then could one talk of project success with respect to time, one may ask.

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Table 4: Executed Projects and their Critical Factors, Bayero University, Kano

S/N	Project	Initial Contract	Final Contract	Contract Variation	% Variation	Initial Date
		Sum	Sum			of Completion
1	Construction and Furnishing of AKTH (1 st Phase)	9066550	10396753.30	1330203.52	14.67%	26 weeks
2	Construction of 2 nd Hostel Block	7691495.19	8053449.54	361954.32		4.70% 10 weeks
3	Construction of Hostel Blocks at AKTH		6584495.19	8080071.15	1495575.96	22.71% 10
4	Construction of 500 Capacity Lecture Hall			92334928.65	92334928.65	
5	Construction of Computer Centre				88601292.10	103651380.15
6	Construction of Students' Hostel Block	20947500.00	21929039.97	981539.97	4.69%	16 weeks
7	Construction of Information Centre	33429905.75	44275650.98	10845745.43		32.44% 16
8	Completion of Conference Centre			93383010.26	96300050.60	2917040.34
9	Completion of Convocation Square				24932617.50	24932617.50
10	Office Block of AKTH	19442428.32	19442428.32			30 weeks

S/N	Final Date of Completion	Period	% of Period Variation	Remarks/ State of Status
1	32 weeks	6 weeks	23.08%	100% completed
2	52 weeks	42 weeks	420.00%	100% completed
3	52 weeks	42 weeks	420.00%	100% completed
4	136 weeks	80 weeks	142.00%	95% completed
5	64 weeks	28 weeks	77.78%	100% completed
6	29 weeks	13 weeks	81.25%	100% completed
7	52 weeks	36 weeks	225.00%	100% completed
8	20 weeks	8 weeks	66.67%	100% completed
9	40 weeks	4 weeks	11.11%	97% completed
10	34 weeks	4 weeks	13.33%	97% completed
11	76 weeks	6 weeks	8.57%	98% completed
12	32 weeks	28 weeks	700.00%	95% completed

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The introduction of the discounting technique was done with an additional purpose of highlighting the value added at different points and its effects on the real cost of production in line with emerging production techniques. Even though the constrained-resource optimization models may be following the philosophy of Just-in-time (Ventura *et al* (2002), Chandra *et al* (1998)) and Supply Chain Management (Gunasekaran *et al* (2004), Serve *et al* (2002)), that is just on time at every node, not necessarily completed and kept for the realization of the node, the traditional project management however, is emphasizing so much on the early completion of the different activities after the preceding activities have been completed rather than as late as possible to minimize the real cost. Project success is sometimes judged on the basis of an early completion time, if possible even before the due date. This may not be advantageous to both parties to the contract. The contractor will lose out in terms of interest charges/foregone if the project cannot be handed over till the due date and secondly he may incur extra cost of looking after the project. The burden will shift to the client if he accepts the project before the due date except of course he has an immediate need for it.

7.0 Conclusion

In today's competitive world, projects must be delivered at the right time, within budget and at an acceptable quality level. Time schedule and an anticipated project cost are always difficult to meet. This paper attributes this mainly to methods used in estimating these parameters which are normally assumed in practical situations to be static. Secondly there is no acknowledgement of idle cost which is inevitable in any system even though a lot of efforts have been made to minimize this in the open literature. Using an integrated approach of all the different variables and considering the fact that these two parameters are always probabilistic, time and cost would therefore lie within a range. Five distribution models have been used for this purpose using a hypothetical case study and this analysis is then used to evaluate series of projects with respect to time and cost overruns as to whether these two parameters stay within acceptable ranges. Finally the present study is viewed in the wider context of modern production systems of Just-In-Time (JIT) and Supply Chain Management and questions the rationale of starting non-critical activities at their earliest starting times.

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APPENDIX A

1. Beta Distribution

$$f(x) = \begin{cases} \frac{\chi^{\alpha-1}(1-\chi)^{\beta-1}}{B(\alpha, \beta)} & 0 < \chi < 1 \\ 0 & \text{otherwise} \end{cases} \quad (\alpha, \beta > 0)$$

where α, β are positive. The mean and variance are

$$\mu = \frac{\alpha}{\alpha + \beta} \quad \sigma^2 = \frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)}$$

2. Exponential Distribution

$$f(x) = \begin{cases} \alpha e^{-\alpha x} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

The mean and the variance are

$$\mu = \frac{1}{\alpha} \quad ; \quad \sigma^2 = \frac{1}{\alpha^2}$$

3. Normal Distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2} \quad -\infty < x < \infty$$

where μ and σ are the mean and standard deviation respectively.

4. Uniform Distribution.

$$f(x) = \begin{cases} 1/(b-a) & a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$$

The mean and variance are respectively

$$\mu = \frac{1}{2}(a+b) \quad \sigma^2 = \frac{1}{12}(b-a)^2$$

5. Weibull Distribution

$$f(x) = \begin{cases} abx^{b-1} e^{-ax^b} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

The mean and standard deviation are

$$\mu = a^{-1/b} \Gamma\left(1 + \frac{1}{b}\right) \quad \sigma^2 = a^{-2/b} \left[\Gamma\left(1 + \frac{2}{b}\right) - \Gamma^2\left(1 + \frac{1}{b}\right) \right]$$